Basics of wetting: Young’s equation

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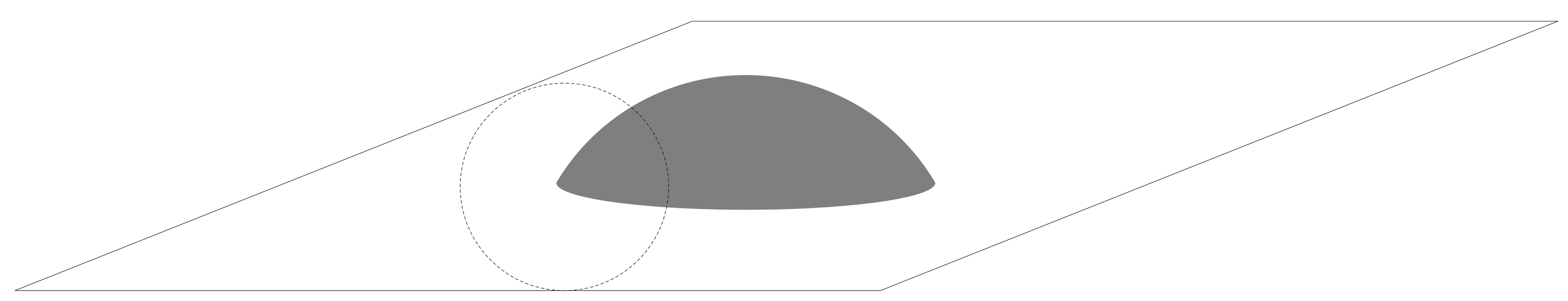
Introduction

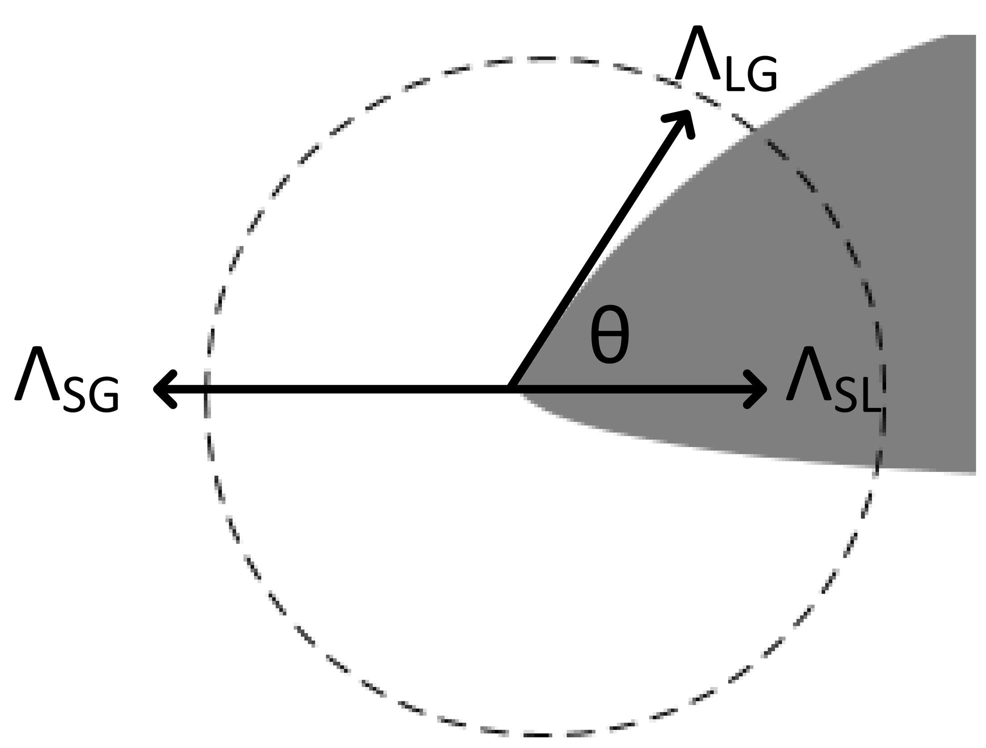
The concept of surface tension and contact angles as a measure for the wetting capability is ascribed to Thomas Young (1804). This led to what we now know as Young’s law. A short derivation will follow here. Let us consider a small drop in the shape of a spherical cap on a wettable surface.



We the have to find the extrema in the total energy of the system. This energy E_s is given by the equation:

$$E_s = (\Lambda_{SL} - \Lambda_{SG})\, \pi\, R^2 (1-u^2) + \Lambda_{LG}\, 2\, \pi\, R^2\, (1-u)$$

where u equals cos*(θ)* and θ is the contact angle*.* The situation near the contact angle is enlarged given below:



We have to find the extema under the constraint that the volume V_{cap} - V_o is zero:

$$V_{cap} - V_o = \frac{2}{3} \pi\, R^3 (1-u) - \frac{1}{3} \pi\, R^3 (1-u^2)\, u - V_o = 0$$

This problem can be solved using Lagrange multipliers:

The objective function Obj(u, R, λ) becomes:

$$\mathrm{Obj}(u, R, \lambda) = E_s - \lambda \left(V_{cap} - V_o\right)$$

By differentiating with regards to u, R and λ and solving the obtained set of equations we obtain the following relation:

$$u = \frac{\Lambda_{SG} - \Lambda_{SL}}{\Lambda_{LG}}$$

which is know as Young’s equation.

The balance of forces in the vertical direction

In the vertical direction the hydrostatic pressure P_{hydr} is balanced by the vertical component of the surface tension:

$$P_{hydr}\, \pi\, f(z)^2 = \gamma\, 2\, \pi\, f(z) \sin(\theta)$$

F(z) describes the contour of the drop as a function of z with respect to the central vertical axis. Using this definition we obtain for sin(θ):

$$\sin(\theta) = \frac{1}{(1 + f'(z)^2)^{1/2}}$$

Upon combining this with the Young-Laplace equation, which apparently firstly was fully written in mathematical form by Gauss we obtain for the force F_{vert} in vertical direction:

$$F_{vert} = \pi\, f(z)^2 \left(-\frac{f''(z)}{(1 + f'(z)^2)^{3/2}} - \frac{1}{f(z)(1 + f'(z)^2)^{1/2}} \right) = 0$$

The wto terms between the brackets are recognized as the two radii of curvature. In simplified form:

$$F_{vert} = \pi\, f(z)^2 \left(\frac{1}{R_1(z)} - \frac{1}{R_2(z)} \right) = 0$$

This implies under absence of external forces the radii of curvature are identical. This in turn has implications for the electronics manufacturing industries where flip chips and BGAs are used. In the latter case the weight of the component deforms the liquid bump and the gravitational force is compensated by the difference in radii of curvature.

$$F_{vert} = \pi\, f(z)^2 \left(\frac{1}{R_1(z)} - \frac{1}{R_2(z)} \right) - M\, g = 0$$

Young’s law and also the Young-Laplace presume that the surface energy is constant over the surface. Therefore one should be aware that deviations may occur in the transition region close to the substrate. Effects like precursor formation, intermetallics and substrate dissolution may affect the contact angle.

According to the later derived Young–Dupré equation (Young 1805; Dupré 1869) neither Λ_{SG} nor Λ_{SL} can be larger than the sum of the other two surface energies. As a consequence complete wetting will occur when Λ_{SG} > Λ_{SL} + Λ_{LG} and no wetting will occur when Λ_{SL} > Λ_{SG} + Λ_{LG} .